RET/13/Test B

904

Mathematics

Day and Da	ite	(Signature of Invigilator)
Serial No. o	of OMR Answer Sheet	
Roll No. (W	rite the digits in words)	***************************************
Roll No.		
	(To be filled up by the candidate by b	ue/black ball-point pen)
	Que	stion Booklet No

INSTRUCTIONS TO CANDIDATES

(Use only blue/black ball-point pen in the space above and on both sides of the Answer Sheet)

- Within 10 minutes of the issue of the Question Booklet, Please ensure that you have got the
 correct booklet and it contains all the pages in correct sequence and no page/question is
 missing. In case of faulty Question Booklet, bring it to the notice of the
 Superintendent/Invigilators immediately to obtain a fresh Question Booklet.
- 2. Do not bring any loose paper, written or blank, inside the Examination Hall except the Admit Card without its envelope.
- 3. A separate Answer Sheet is given. It should not be folded or mutilated. A second Answer Sheet shall not be provided.
- 4. Write your Roll Number and Serial Number of the Answer Sheet by pen in the space provided above.
- 5. On the front page of the Answer Sheet, write by pen your Roll Number in the space provided at the top, and by darkening the circles at the bottom. Also, wherever applicable, write the Question Booklet Number and the Set Number in appropriate places.
- 6. No overwriting is allowed in the entries of Roll No., Question Booklet No. and Set No. (if any) on OMR sheet and Roll No. and OMR sheet No. on the Question Booklet.
- 7. Any changes in the aforesaid-entries is to be verified by the invigilator, otherwise it will be taken as unfair means.
- 8. This Booklet contains 40 multiple choice questions followed by 10 short answer questions. For each MCQ, you are to record the correct option on the Answer Sheet by darkening the appropriate circle in the corresponding row of the Answer Sheet, by pen as mentioned in the guidelines given on the first page of the Answer Sheet. For answering any five short Answer Questions use five Blank pages attached at the end of this Question Booklet.
- 9. For each question, darken only one circle on the Answer Sheet. If you darken more than one circle or darken a circle partially, the answer will be treated as incorrect.
- 10. Note that the answer once filled in ink cannot be changed. If you do not wish to attempt a question, leave all the circles in the corresponding row blank (such question will be awarded zero marks).
- 11. For rough work, use the inner back page of the title cover and the blank page at the end of this Booklet.
- 12. Deposit both OMR Answer Sheet and Question Booklet at the end of the Test.
- 13. You are not permitted to leave the Examination Hall until the end of the Test.
- 14. If a candidate attempts to use any form of unfair means, he/she shall be liable to such punishment as the University may determine and impose on him/her.

[उपर्युक्त निर्देश हिन्दी में अन्तिम आवरण-पृष्ठ पर दिये गये हैं।]

Total No. of Printed Pages: 20

FOR ROUGH WORK/रफ कार्य के लिए

Research Entrance Test - 2013

No. of Questions: 50

प्रश्नों की संख्या : 50

Time: 2 Hours

Full Marks: 200

समय : 2 घण्टे

पूर्णांक : 200

Note: (i) This Question Booklet contains 40 Multiple Choice Questions followed by 10 Short Answer Questions.

इस प्रश्न पुस्तिका में 40 वस्तुनिष्ठ व 10 लघु उत्तरीय प्रश्न हैं।

(ii) Attempt as many MCQs as you can. Each MCQ carries 3 (Three) marks. 1 (One) mark will be deducted for each incorrect answer. Zero mark will be awarded for each unattempted question. If more than one alternative answers of MCQs seem to be approximate to the correct answer, choose the closest one.

अधिकाधिक वस्तुनिष्ठ प्रश्नों को हल करने का प्रयत्न करें। प्रत्येक वस्तुनिष्ठ प्रश्न 3 (तीन) अंकों का है। प्रत्येक गलत उत्तर के लिए 1 (एक) अंक काटा जायेगा। प्रत्येक अनुत्तरित प्रश्न का प्राप्तांक शून्य होगा। यदि वस्तुनिष्ठ प्रश्नों के एकाधिक वैकल्पिक उत्तर सही उत्तर के निकट प्रतीत हों, तो निकटतम सही उत्तर दें।

(iii) Answer only 5 Short Answer Questions. Each question carries 16 (Sixteen) marks and should be answered in 150-200 words. Blank 5 (Five) pages attached with this booklet shall only be used for the purpose. Answer each question on separate page, after writing Question No.

केवल 5 (पाँच) लघु उत्तरीय प्रश्नों के उत्तर दें। प्रत्येक प्रश्न 16 (सोलह) अंकों का है तथा उनका उत्तर 150-200 शब्दों के बीच होना चाहिए। इसके लिए इस पुस्तिका में लगे हुए सादे 5 (पाँच) पृष्ठों का ही उपयोग आवश्यक है। प्रत्येक प्रश्न का उत्तर एक नए पृष्ठ से, प्रश्न संख्या लिखकर शुरू करें।

1.	Who among the fol	lowi	ng is a Sanskrit	Maha	akavi ?		
	(1) Ashvaghosh	(2)	Keshavadas	(3)	Bihari	(4)	Mammata
	निम्नलिखित में से कौन	संस्वृ	त महाकवि हैं ?	,			
	(1) अश्वघोष	(2)	केशवदास	(3)	बिहारी	(4)	मम्मट
	20 P. T				XII		
2.	Who collected the p	oem	s of Gahasatta	sai?			
	(1) Bilhana	(2)	Kalhana	(3)	Udbhata	(4)	Haala
	<i>'गाहासत्तसई'</i> इस काव्य	गग्रन्थ	के संग्रहकर्ता कौन	7 考?			
	(1) बिल्हण	(2)	कल्हण	(3)	उद्भट	(4)	हाल
3.	Who among the fo	Mou	ting introduced	l the	theory of Kath	arci	e in the western
٥.	poetics?	JIIOV	ing introduced	i tile	meory of Run	ursi	s in the western
	(1) Socrates	(2)	Aristotle	(3)	Colaridge	(4)	T. S. Eliot
	पाश्चात्य काव्यशास्त्र में	33 Ni			-		
	(1) सुकरात		अरस्तू		39		टी॰ एस॰ इलियट
	(3) 3,	(-)		(-)		\ /	96000 № 9500 end \$0,0000 80 50
4.	Which of the follow	ving	is the work of f	amoı	ıs Egyptian criti	c and	d poet Hores?
	(1) Peri Poetics			(2)	Ars Poetica	155	
	(3) Biographia Lite	erari	a	(4)	Capital		
	निम्नलिखित में से कौन	मिरु	के प्रसिद्ध समालो	चक त	ाथा कवि होरेस की	कृति	है ?
	(1) पेरी पोएटिक्स			(2)	आर्स पोएटिका		
	(3) बायोग्राफिया लिटरे	रिया	į	(4)	केपिटल		
200-2							8
5.	Ishavasyopnishad						Camalaira
	(1) Shukla Yajurye	eds S	amhita		Krishna Yajury Samaveda	veda	Samnita
	(3) Rigveda ईशावास्योपनिषद निम्न	ं बिलिस्	रत में से किस वेद		www. 100.00		
	(1) शुक्लयजुर्वेद संहित				कृष्णयजुर्वेद संहित	11	
	(3) ऋग्वेद				सामवेद	C. (4)	
RET/	13/Test B/904		(2)	<u>ģ</u>	1.8	:2

	8	
6.	Shabarbhasya is the book related to wh	ich philosophy?
	(1) Nyaya (2) Yoga	(3) Mimamsa (4) Vaishesika
	शाबरभाष्य किस दर्शन का ग्रन्थ है ?	· landing
	(1) न्याय (2) योग	(3) मीमांसा (4) वैशेषिक
7.	Who propounded the theory of Vishisht	advaitavads and Dvaitadvaitvada?
22	(1) Ramanuja and Nimbarka	CONTRACTOR OF THE PARTY AND ADDRESS OF
	(3) Ramanuja and Vallabha	(4) Nimbarka and Ramanuja
	निम्नलिखित में कौन क्रमशः विशिष्टाद्वैतवाद तथा	
	(1) रामानुज तथा निम्बार्क	(2) निम्बार्क तथा मध्याचार्य
	, , ,	(4) निम्बार्क और रामानुज
	(3) रामानुज और वल्लभ	(4) निन्दाक अरि रानानुग
. 8.	Who wrote the commentary Mitakshara	on Yainavalkuasmiriti ?
, 0.	(1) Vijnanabhikshu (2) Vijnanesvara	
	याझवल्क्यरमृति पर <i>'मिताक्षरा'</i> टीका के रचयिता	
		(3) विज्ञानसूरि (4) यास्क
	(1) विज्ञानभिक्षु (2) विज्ञानेश्वर	(3) विशानसूरि (4) वरिक
9.	Select the correct order:	
3.	सही मिलान चुनिये :	건글 벡
	1. Pratimanatak/प्रतिमानाटक	(क) Vishakhadatta/विशाखदत्त
	2. Mudrarakshasa/मुद्राराक्षस	(ख) Bhattanarayana/भट्टनारायण
	3. Venisamhara/वेणीसंहार	(ग) Shudraka/शूद्रक
	4. Mrichchhakatika/मृच्छकटिक	(घ) Bhasa/भास
	(1) 1 (ঘ) 2 (क) 3 (ख) 4 (ग)	(2) 1 (क) 2 (ख) 3 (ग) 4 (ঘ)
	(3) 1 (क) 2 (घ) 3 (ख) 4 (ग)	(4) 1(日) 2(西) 3(刊) 4(四)
	(3) 1 (4) 2 (4) 3 (4) 1 (1)	(1) 1 (1) 2 (1) 3 (1) 1 (1)
10.	Complete name of Urdu and Persian po	oet 'Ghalib' is :
		(2) Asadullah Khan Ghalib
	(1) Mirza Asadullah Khan Ghalib	100 ACC 101 AC
	(1) Mirza Asadullah Khan Ghalib	(4) Ghalib Khan Asadullah
	(1) Mirza Asadullah Khan Ghalib(3) Mirza Khan Asadullah Ghalib	(4) Ghalib Khan Asadullah ਜਾਸ हੈ :
	(1) Mirza Asadullah Khan Ghalib (3) Mirza Khan Asadullah Ghalib उर्दू तथा फ़ारसी भाषा के कवि 'गालिब' का पूरा	(4) Ghalib Khan Asadullah ਜਾਸ हੈ :

11. The radius of convergence of the power series,

$$1 + \frac{a \cdot b}{1 \cdot c}z + \frac{a(a+1)b(b+1)}{1 \cdot 2c(c+1)}z^2 + \cdots$$

where a,b,c being complex numbers is:

- (1) 1
- (2) 2
- (3) 3
- (4) 4

12. The function $f(z) = \sin\left\{z\left(z + \frac{1}{z}\right)\right\}$ has been expanded in the following series,

$$f(z) = a_0 + \sum_{n=1}^{\infty} a_n \left(z^n + \frac{1}{z^n} \right),$$

where a_n , $n \ge 1$, are given by :

(1)
$$a_n = \frac{1}{2\pi} \int_0^{2\pi} \sin(2\cos\theta) \cos n\theta \, d\theta$$

(2)
$$a_n = \frac{1}{2\pi} \int_0^{2\pi} \sin(4\cos\theta) \cos n\theta \, d\theta$$

(3)
$$a_n = \frac{1}{2\pi} \int_0^{2\pi} \sin(\cos\theta) \cos n\theta \, d\theta$$

(4)
$$a_n = \frac{1}{2\pi} \int_0^{2\pi} \sin(4\sin\theta) \cos n\theta \, d\theta$$

13. Which of the following subsets of real numbers \mathbb{R} does not have the Cardinal number C?

- (1) $\{x \in \mathbb{R} : 0 < x < 1\}$
- (2) $\{x \in \mathbb{R} : 0 \le x < 2\}$
- (3) $\{x \in \mathbb{R} : x \ge 0\}$
- (4) $\{x : x \in \mathbb{N} = \text{Set of natural numbers} \}$

- **14.** For a measure space (X, s, μ) , which of the following statements is not true?
 - (1) If $\{A_n\}_{n=1}^{\infty}$ is a sequence of pairwise disjoint measurable subsets of X, then

$$\mu\left(\bigcup_{n=1}^{\infty}A_{n}\right)=\sum_{n=1}^{\infty}\mu(A_{n})$$

- (2) $\mu(\phi) = 0$ where ϕ is a null set
- (3) If $A, B \in s$, $A \subseteq B$, then $\mu(A) \le \mu(B)$
- (4) If $\{B_n\}_{n=1}^{\infty}$ is a sequence of subsets of X, then

$$\mu\left(\bigcup_{n=1}^{\infty}B_n\right)=\sum_{n=1}^{\infty}\mu(B_n)$$

15. If $\sum_{n=0}^{\infty} a_n$ and $\sum_{n=0}^{\infty} b_n$ are each convergent series of real numbers then their

Cauchy's product series $\sum_{n=0}^{\infty} c_n$, where

$$c_n = \sum_{k=0}^n a_k b_{n-k}, \quad n \ge 0, \text{ is :}$$

- (1) convergent but not absolutely convergent,
- (2) absolutely convergent
- (3) not convergent
- (4) divergent if $\sum_{n=0}^{\infty} a_n$ is absolutely convergent
- 16. Which of the following statements is not true?
 - (1) If X is a non-empty set and P(X) is a power set of X then (X, P(X)) is a topological space
 - (2) If X is a non-empty set and $T = \{\phi, X\}$ then $\{X, T\}$ is a topological space
 - (3) If X is an infinite set and $T = \{Y \subseteq X : Y \text{ is an empty set or } Y' \text{ is finite}\}$, then (X, T) is a topological space
 - (4) If $X = \{a, b, c\}$ and $T = \{\phi, X, \{a, b\}, \{a, c\}\}\$, then (X, T) is a topological space

	(1) 5	** i	(2) 1	6	(3) 20		(4) 24	
18.							ere $\mathbb{Z}[i]$ is the rinerated by $2+i$?	ıg
	(1) 2	1. 3	(2) 5	į	(3) 7		(4) infinite	
19.	What is thabelian gr	ne smalle: oups of o	st posi rder <i>n</i>	tive integer <i>n</i>	such that	there are	four non-isomorph	ic
	(1) 24	31 25	(2) 3	6	(3) 48		(4) 64	
20.	Which of $\mathbb{Z}[i]$:	the follo	wing s	statement is c	orrect ? In	the ring	of Gaussian intege	rs
	(1) 3 and	2 are irre	ducibl	e elements, bu	ıt 5 is not i	rreducible		
	(2) 3 and	5 are irre	ducible	e elements, bu	it 2 is not i	rreducible		
	(3) 3 is an	irreduci	ble ele	ment but 2 an	d 5 are not	irreducibl	e ·	
	(4) 5 is ar	ı irreduci	ble ele	ment but 3 an	d 2 are not	irreducibl	e	
21.	The degree		splitt	ing field of t	he polyno	mial x ⁶ +	1 over the field	of
	(1) 3	<i>\$0</i>	(2) 8	*	(3) 6		(4) 4	
22.	If A is a r	ilpotent i	natrix,	, then A is sin	nilar to :			
	(1) a diag							
	(2) a scala	ar matrix						86
£	(3) a triar	ngular ma	ıtrix					
	(4) a triar	ngular ma	itrix w	hose entries o	n the main	diagonal a	are all zero	
23.	If the mo-		fluid i	is irrotational	, then the	velocity v	ector \vec{q} satisfies th	ıe
	(1) div \vec{q}	= 0	(2) d	iv $\vec{q} \neq 0$	(3) curl <i>i</i>	$\vec{q} = 0$	(4) curl $\vec{q} \neq 0$	
RET/	13/Test B/9	04	196	(6)		8		

17. The number of elements of order 5 in the group $\mathbb{Z}_{25} \oplus \mathbb{Z}_5$ are :

The degree of freed (1) 1	om of a rigid body (2) 2	in motion with (3) 3	its two points fixed is: (4) 4			
rigid body can be re of the couple coinci	educed to a single des with the direct	force and a sing ion of the force,	tle couple in which the ax then the line is called:	f a kis		
3.0		2	N			
Among all the curv which generates the	es joining two give surface of minim	en points $A(x_0)$ um area when r	y_0) and $B(x_1, y_1)$, the o otated about the x -axis is	ne s :		
(1) cycloid	(2) parabola					
throughout the mo	tion:	¥).	8	en.		
(2) both its kinetic	(2) both its kinetic energy and angular momentum vary					
(3) its kinetic energ	(3) its kinetic energy varies but the angular momentum remains constant					
(4) both its kinetic	energy and angula	ii momentum r	131/2 =			
	formulation of a	dynamical syste	em, the equations of moti	ion		
(2) one set of first order differential equations						
(3) two sets of second order differential equations						
(4) two sets of firs	t order differential	equations	~ .			
The integral equati	on given by : $u(x) = f(x)$	$+\int_a^b k(x,\xi)u(\xi)a$	<i>ا</i> لإ	6		
is known as:			8			
			·			
13/Test B/904			P. 1	T. O.		
	If at any point of a rigid body can be re of the couple coinci. (1) Pitch Among all the curv which generates the (1) cycloid If a rigid body is rethroughout the mod. (1) its kinetic energ. (2) both its kinetic. (3) its kinetic energ. (4) both its kinetic. In the Hamiltonian are in terms of: (1) one set of second. (2) one set of first. (3) two sets of second. (4) two sets of first. (5) the integral equations is known as: (1) Volterra integral. (2) Fredholm integral. (3) Volterra integral. (4) Fredholm integral.	If at any point of a straight line, a systrigid body can be reduced to a single of the couple coincides with the direct (1) Pitch (2) Central axis Among all the curves joining two give which generates the surface of minimum (1) cycloid (2) parabola If a rigid body is rotating about one of throughout the motion: (1) its kinetic energy remains constant (2) both its kinetic energy and angulation (3) its kinetic energy varies but the art (4) both its kinetic energy and angulation of a care in terms of: (1) one set of second order differentiation (2) one set of first order differentiation (3) two sets of second order differentiation (4) two sets of first order differential (5) two sets of first order differential (6) two sets of first order differential (7) the integral equation given by: $u(x) = f(x)$ is known as: (1) Volterra integral equation of the first order differential equation of the first order integral equation of the first order differential (4) Fredholm integral equation of the first order differential (5) Volterra integral equation of the first order differential (6) Fredholm integral equation of the first order differential (7) Fredholm integral equation of the first order differential (8) Volterra integral equation of the first order differential (9) Fredholm integral equation of the first order differential e	If at any point of a straight line, a system of forces are rigid body can be reduced to a single force and a single of the couple coincides with the direction of the force, (1) Pitch (2) Central axis (3) Wrench Among all the curves joining two given points $A(x_0, x_0)$, which generates the surface of minimum area when refused throughout the surface of minimum area when refused throughout the motion: (1) its kinetic energy remains constant but the angular (2) both its kinetic energy and angular momentum version is kinetic energy varies but the angular momentum version in terms of: (1) one set of second order differential equations (2) one set of first order differential equations (3) two sets of second order differential equations (4) two sets of first order differential equations (5) the integral equation given by: $u(x) = f(x) + \int_a^b k(x, \xi) u(\xi) d\xi$ is known as: (1) Volterra integral equation of the first kind (2) Fredholm integral equation of the second kind (4) Fredholm integral equation of the second kind (4) Fredholm integral equation of the second kind (4) Fredholm integral equation of the second kind (5) Fredholm integral equation of the second kind (6) Fredholm integral equation of the second kind (7)	If at any point of a straight line, a system of forces acting at different points or rigid body can be reduced to a single force and a single couple in which the are of the couple coincides with the direction of the force, then the line is called: (1) Pitch (2) Central axis (3) Wrench (4) Screw Among all the curves joining two given points $A(x_0, y_0)$ and $B(x_1, y_1)$, the owhich generates the surface of minimum area when rotated about the x -axis is (1) cycloid (2) parabola (3) hyperbola (4) catenary If a rigid body is rotating about one of its own fixed point under no forces, the throughout the motion: (1) its kinetic energy remains constant but the angular momentum varies (2) both its kinetic energy and angular momentum vary (3) its kinetic energy varies but the angular momentum remains constant (4) both its kinetic energy and angular momentum remains constant In the Hamiltonian formulation of a dynamical system, the equations of motion are in terms of: (1) one set of second order differential equations (2) one set of first order differential equations (3) two sets of second order differential equations (4) two sets of first order differential equations The integral equation given by: $u(x) = f(x) + \int_0^b k(x, \xi) u(\xi) d\xi$ is known as: (1) Volterra integral equation of the first kind (2) Fredholm integral equation of the second kind (4) Fredholm integral equation of the second kind		

	(3) Laplace equation	(4) None of these
34.	If a function $f: \mathbb{R}^2 \to \mathbb{R}$ is defined as	<u>.</u>
	f(x) =	$2x_1x_2-x_1^2-x_2^2$,
	then which of the following is true	for the function f :
	(1) Its Hessian is positive definite h	nence is convex
	(2) Its Hessian is positive semi-defi	inite hence is concave

(3) Its Hessian is negative definite hence is convex

(4) Its Hessian is negative semi-definite hence is concave

RET/13/Test B/904

30.

31.

32.

33.

is:

(1).5

(1) 3

For the matrix

(1) Wave equation

The differential equation

(1) Legendre equation

(3) Hypergeometric equation

the value of row norm $||A||_{\infty}$ is:

where n is a constant, is known as:

The maximum number of basic solutions of the system,

 $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 5 \\ 3 & 5 & 1 \end{bmatrix}$

(2) 6

(2) 5

In finite difference method, the five point formula is used for solution of:

 $(1-x^2)y'' - 2xy' + n(n+1)y = 0,$

 $2x_1 - 5x_2 + x_3 + 3x_4 = 4$

 $3x_1 - 10x_2 + 2x_3 + 6x_4 = 12$

(3) 7

(2) Heat equation

(2) Bessel equation

(4) Laguerre equation

(4) 8

(4) 9

35. Suppose that a curve $\alpha : [0, 4] \to \mathbb{R}^3$ is given by

$$\alpha(t) = \left(t - 3, \sqrt{t}, t\sqrt{t}\right)$$

If $\phi:[0,2] \rightarrow [0,4]$ is given by

$$\phi(u)=u^2\,,$$

then the reparametrization β of the curve α by the map ϕ is :

(1)
$$\beta(u) = (u^2 + 3, u^2, -3u^2)$$

(2)
$$\beta(u) = (u^2 - 3, u, u^3)$$

(3)
$$\beta(u) = (u^2 - 3, u^2, u^3)$$

(4)
$$\beta(u) = (u, u^2, u^3)$$

36. Let $\alpha(s) = (r \cos ws, r \sin ws, hws)$ be a unit speed circular helix with $w = \frac{1}{\sqrt{r^2 + h^2}}$.

then:

(1)
$$k(s) = w^2 r$$
, $\tau(s) = h^2 r$

(2)
$$k(s) = r^2 w$$
, $\tau(s) = h^2 w$

(3)
$$k(s) = w^2 r$$
, $\tau(s) = w^2 h$

(4)
$$k(s) = h^2 r$$
, $\tau(s) = h^2 w$

37. Which of the following statement is true for the cylinder?

$$C^2 = \{(x, y, z) \in \mathbb{R}^3 | x^2 + y^2 = 1 \}$$

and the sphere

$$S^2 = \{(x, y, z) \in \mathbb{R}^3 | x^2 + y^2 + z^2 = 1 \}$$
?

- (1) C^2 and S^2 cannot be covered by a single surface patch
- (2) C^2 and S^2 can be covered by a single surface patch
- (3) C^2 cannot be covered by a single surface patch but S^2 can be covered by a single surface patch
- (4) C^2 can be covered by a single surface patch but S^2 cannot be covered by a single surface patch

38. The set of all straight lines in the Euclidean plane is a C^{∞} manifold of dimension:

RET/13/Test B/904

(9)

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39. Consider the function

$$\alpha: \mathbb{R} \to \mathbb{R}^2$$

given by

$$\alpha(t) = \left(t^3 - 4t, t^2 - 4\right)$$

then:

- (1) α is an embedding with self intersection for t=-2 and t=2, but α is not an immersion
- (2) α is an immersion but it is not an embedding
- (3) α is a submersion but not an immersion
- (4) α is an embedding but not a submersion
- **40.** Let $f:M_1 \to M_2$ and $g:M_2 \to M_3$ be C^{∞} maps on C^{∞} manifolds. Then the Jacobian maps of f and $g:M_2 \to M_3$ be C^{∞} maps on C^{∞} manifolds.
 - (1) are linear and satisfy the chain rule
 - (2) are linear but fail to satisfy the chain rule
 - (3) are non-linear and satisfy the chain rule
 - (4) are non-linear but fail to satisfy the chain rule

Attempt any five questions. Write answer in 150-200 words. Each question carries 16 marks. Answer each question on separate page, after writing Question Number. किन्हीं पाँच प्रश्नों के उत्तर 150-200 शब्दों में दीजिए। प्रत्येक प्रश्न 16 अंकों का है। प्रत्येक प्रश्न का उत्तर अलग पृष्ठ पर प्रश्न संख्या लिखकर शुरू करें।

- Show that a subset of a topological space is closed if and only if it contains its boundary.
- **2.** Prove that B, B' are Banch spaces over $k (= \mathbb{R} \text{ or } \mathbf{C})$ and T is a linear transformation of B into B' then T is continuous \iff its graph is closed.
- **3.** Show that any field F of characteristic zero is a perfect field.
- **4.** If N is a submodule of an R-module M such that N and M/N both are noetherian then show that M is noetherian.
- 5. Verify whether or not the transformation

$$P = \frac{1}{2}(p^2 + q^2), Q = \tan^{-1}\left(\frac{q}{p}\right)$$

is a contact transformation?

6. For an incompressible homogeneous fluid, the velocity distribution at the point (x, y, z) is given by

$$u = -\left(c^2y/r^2\right),$$

$$v = \left(c^2 x / r^2\right),$$

$$w = 0$$

Where r denotes the distance from the z-axis. Show that it is a possible motion and determine the surface which is orthogonal to stream-lines.

7. Reduce the initial value problem

$$\frac{d^2y}{dx^2} = \lambda y(x) + g(x)$$

$$y(0) = 1, y'(0) = 0$$

into Volterra integral equation of the second kind.

- Prove that the solutions of the Dirichlet problem depend continuously on the boundary data.
- 9. Consider the sphere

$$S^{2}(1) = \left\{ \left(x^{1}, x^{2}, x^{3}\right) \in \mathbb{R}^{3} / \left(x^{1}\right)^{2} + \left(x^{2}\right)^{2} + \left(x^{3}\right)^{2} = 1 \right\}$$

consider the collection

$$A = \{(\phi_N, U_N), (\phi_S, U_S)\},\$$

where

$$U_N = S^2(1) - \{(0, 0, 1)\},\$$

$$U_S = S^2(1) - \{(0, 0, -1)\}$$

and

$$\phi_N: U_N \to \mathbb{R}^2 \text{ and } \phi_S: U_S \to \mathbb{R}^2$$

are the stereographic projections. Then prove that A is an atlas for $S^2(1)$.

10. Let M be a Riemannian manifold with its Riemannian connection ∇ . Then prove that the Hessian Hf of a C^{∞} function f on M is given by

$$(Hf)(X,Y) = \langle \nabla_X (grad f), Y \rangle, X, Y \in TM$$

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FOR ROUGH WORK/रफ कार्य के लिए

अभ्यर्थियों के लिए निर्देश

(इस पुस्तिका के प्रथम आवरण-पृष्ट पर तथा *उत्तर-पत्र* के दोनों पृष्ठों पर केवल *नीली।काली बाल-प्वाइंट पेन* से ही लिखें)

- 1. प्रश्न पुस्तिका मिलने के 10 मिनट के अन्दर ही कृपया देख लें कि प्रश्नपत्र में सभी पृष्ठ मौजूद हैं और कोई प्रश्न छूटा नहीं है। पुस्तिका दोषयुक्त पाये जाने पर इसकी सूचना तत्काल कक्ष निरीक्षक को देकर सम्पूर्ण प्रश्नपत्र की दूसरी पुस्तिका प्राप्त कर लें।
- 2. परीक्षा भवनं में *लिफाफा रहित प्रवेश-पत्र के अतिरिक्त*, लिखा या सादा कोई भी खुला कागज साथ में न लायें।
- 3. उत्तर-पत्र अलग से दिया गया है। इसे न तो मोड़ें और न ही विकृत करें। दूसरा उत्तर-पत्र नहीं दिया जायेगा।
- 4. अपना अनुक्रमांक तथा उत्तर-पत्र का क्रमांक प्रथम आवरण-पृष्ट पर पेन से निर्धारित रशान पर लिखें।
- 5. उत्तर-पत्र के प्रथम पृष्ठ पर पेन से अपना अनुक्रमांक निर्धारित स्थान पर लिखें तथा नीचे दिये वृत्तों को गाढ़ा कर दें। जहाँ-जहाँ आवश्यक हो वहाँ प्रश्न-पुरितका का क्रमांक तथा सेट का नम्बर उचित स्थानों पर लिखें।
- 6. ओ॰ एम॰ आर॰ पत्र पर अनुक्रमांक संख्या, प्रश्न-पुस्तिका संख्या व सेट संख्या (यदि कोई हो) तथा प्रश्न-पुस्तिका पर अनुक्रमांक संख्या और ओ॰ एम॰ आर॰ पत्र संख्या की प्रविष्टियों में उपरिलेखन की अनुमति नहीं है।
- 7. उपर्युक्त प्रविष्टियों में कोई भी परिवर्तन कक्ष निरीक्षक द्वारा प्रमाणित होना चाहिये अन्यथा यह एक अनुचित साधन का प्रयोग माना जायेगा।
- 8. प्रश्न-पुस्तिका में 40 बहुविकल्पीय तथा 10 लघु उत्तरीय प्रश्न हैं। प्रत्येक बहुविकल्पीय प्रश्न के चार वैकल्पिक उत्तर दिये गये हैं। प्रत्येक प्रश्न के वैकल्पिक उत्तर के लिये आपको उत्तर-पत्र की सम्बन्धित पंक्ति के सामने दिये गये वृत्त को उत्तर-पत्र के प्रथम पृष्ठ पर दिये गये निर्देशों के अनुसार पेन से गाढ़ा करना है। किन्हीं पाँच लघु उत्तरीय प्रश्नों के उत्तर के लिए प्रश्न पुस्तिका के अन्त में पाँच खाली पृष्ठ दिये गये हैं।
- 9. प्रत्येक प्रश्न के उत्तर के लिये केवल एक ही वृत्त को गाढ़ा करें। एक से अधिक वृत्तों को गाढ़ा करने पर अथवा एक वृत्त को अपूर्ण भरने पर वह उत्तर गलत माना जायेगा।
- 10. ध्यान दें कि एक बार स्याही द्वारा अंकित उत्तर बदला नहीं जा सकता है। यदि आप किसी प्रश्न का उत्तर नहीं देना चाहते हैं, तो सम्बन्धित पंक्ति के सामने दिये गये सभी वृत्तों को खाली छोड़ दें। ऐसे (प्रश्नों पर शून्य अंक दिये जायेंगे।)
- 11. रफ कार्य के लिये इस पुस्तिका के मुखपृष्ठ के अंदर वाला पृष्ठ तथा अंतिम खाली पृष्ठ का प्रयोग करें।
- 12. परीक्षा के उपरान्त दोनों ओ० एम० आर० उत्तर-पत्र एवं प्रश्न पुरितका परीक्षा भवन में जमा करें।
- 13. परीक्षा समाप्त होने से पहले परीक्षा भवन से बाहर जाने की अनुमति नहीं होगी।
- 14. यदि कोई अभ्यर्थी परीक्षा में अनुचित साधनों का प्रयोग करता है, तो वह विश्वविद्यालय द्वारा निर्धारित दंड का / की भागी होगा / होगी।